Applications—Polaroids are cheaper than Nicols and are used in the lab. for the production and analysis of plane-polarised light.

They are also widely used in *sun-glasses* to cut off the glare of light from horizontal surfaces. K-polaroids are extensively used in *head-lights* and *wind screens* of cars to cut-off the dazzling light of an approaching car from opposite direction. They are also employed in the *windows of trains and planes* to control the intensity of light entering into them and *in viewing stereoscopic motion pictures.*

15.11 Huygens' theory of double refraction

According to Huygens' wave theory, each point on a wavefront becomes the source of new disturbance and send secondary wavelets. The envelope of all the secondary wavelets at any instant gives the new wavefront which is the locus of particles vibrating in the same phase. *Huygens' theory in its original form cannot explain double refraction in uniaxial crystals.* He therefore extended his theory of the secondary wavelets to explain the phenomenon. We give below his extended theory in a *summary fashion*, emphasising only on its ***salient features.***

1. When a wavefront is incident on a doubly refracting crystal, every point on the crystal becomes the origin of *two wavefronts*, O and E—one the *ordinary* and the other *extraordinary*—to account for the two types of rays.

2. Ordinary wavefront corresponds to the ordinary rays which obey the laws of refraction and have the *same velocity in all directions.* The *ordinary wavefront is* thus ***spherical.***

3. Extraordinary wavefront corresponds to the extraordinary rays which do not obey the laws of refraction and have *different velocities in different directions.* The extraordinary *wavefront* is thus an *ellipsoid of revolution*, with the optic axis as the axis of revolution.

4. The sphere and the ellipsoid touch each other at points that lie on the optic axis of the crystal, since the *velocity of the ordinary and the extraordinary ray is the same along the optic axis.* So, the uniaxial crystals *do not* show any *double refraction* along the optic axis.

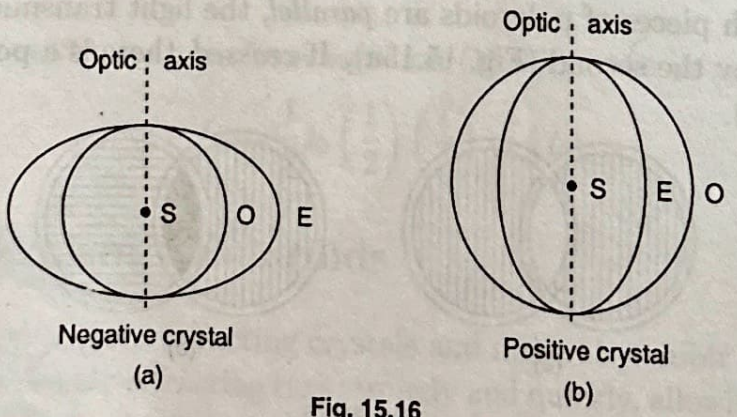

Fig. 15.16
Uniaxial crystals : (a) Negative crystal, (b) Positive crystal

5. In crystals e.g., calcite and tourmaline, called the ***negative crystals***, the ellipsoid lies outside the sphere O, S being the source (Fig. 15.16a), implying that in such crystals the *extraordinary wavefront travels faster* than the ordinary wavefront, except along the optic axis (O.A.).

In some other crystals, e.g., quartz and ice, called the ***positive crystals***, the sphere O is outside the ellipsoid E (Fig. 15.16b), implying that in such crystals the *ordinary wavefront travels faster* than the extraordinary wavefront, except however along O.A.

15.12 Huygens' construction of wave surfaces (uniaxial crystal)

For construction of wave surfaces on the plane of incidence in uniaxial crystals we discuss here *three* distinctly different cases.

Case 1. Optic axis inclined to upper face but lying in the plane of incidence—Let a plane wavefront AB be incident obliquely on the upper face of a *uniaxial negative crystal*, say calcite, cut such that the optic axis lies in the plane of incidence along AX, but inclined to the upper face at some angle (Fig. 15.17).

The point A of the wavefront AB, where it first strikes the crystal surface becomes the centre of both ordinary and extraordinary wavelets O and E. By the time the disturbance from B reaches C, the ordinary wavefront travels a distance AD and the extraordinary wavefront travels over AF.

$$\therefore \quad \frac{BC}{c} = \frac{AD}{v_0} = \frac{AF}{v_e}$$

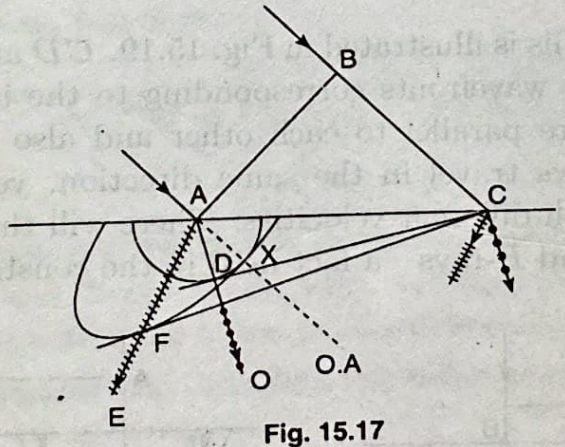

Fig. 15.17
Refraction of a plane wave incident obliquely on a negative crystal with O.A. in the plane of incidence

where c = light velocity in air, v_0 = velocity of O-ray in the crystal and v_e = velocity of the extraordinary light in the crystal.

$$\therefore \quad AD = BC(v_0/c) = BC/n_0;$$

$$AF = BC(v_e/c) = BC/n_e$$

n's being the indices for the two types of rays.

• In both the cases, the E-ray moves through the prism parallel to the base and perpendicular to the optic axis. The very condition used in the definition of n_e is satisfied in the determination.

• The quantity $\Delta n = (n_e - n_0)$ is often called the ***birefringence*** and a uniaxial crystal is termed *positive* or *negative* depending on the sign of Δn.

15.14 Superposition of two plane polarised waves with vibrations at right angles

Let a plane polarised monochromatic light be incident *normally* on a *calcite crystal, cut with optic axis parallel to its faces* and A be the amplitude of vibration that makes an angle θ with optic axis so that the vibration is along PA (Fig. 15.23).

On entering the crystal at P, the light breaks up into (i) O-wave of amplitude $b = A\sin\theta$ with vibrations normal to optic axis and (ii) E-wave of amplitude $a = A\cos\theta$ with vibrations along the optic axis. The O- and E-waves tavel in the crystal in the *same direction* but with *different velocities.* On emergence, therefore, a path difference $d(n_0 - n_e)$ would occur between them, where d is the thickness of the crystal and n_0, n_e are the indices of O-ray and E-ray for the crystal. Calcite is a *negative* crystal and so $n_0 > n_e$.

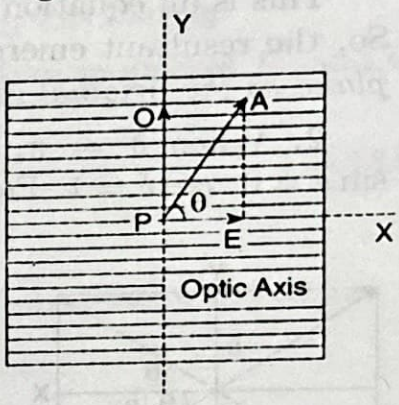

Fig. 15.23
Superposition of two plane polarised light with vibrations at right angles

$$\therefore \quad \text{Phase difference, } \delta = \frac{2\pi}{\lambda} d(n_0 - n_e)$$

So, the equation of E- and O-waves may be written as

$$x = A\cos\theta \sin(\omega t + \delta) = a\sin(\omega t + \delta) \tag{15.14.1}$$

$$y = A\sin\theta \sin\omega t = b\sin\omega t \tag{15.14.2}$$

where ω = the frequency of vibration.

From (15.14.1), we obtain

$$\frac{x}{a} = \sin(\omega t + \delta) = \sin\omega t\cos\delta + \cos\omega t\sin\delta$$

$$= \frac{y}{b}\cos\delta + \sqrt{1 - \frac{y^2}{b^2}}\sin\delta, \quad \text{using (15.14.2)}$$

$$\Rightarrow \left(\frac{x}{a} - \frac{y}{b}\cos\delta\right)^2 = \left(1 - \frac{y^2}{b^2}\right)\sin^2\delta$$

$$\therefore \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta} \tag{15.14.3}$$

which, in general, is the equation of *an ellipse* confined in a rectangle of sides $2a$ and $2b$. The nature of the resultant emerging light however depends on the value of the phase difference δ.

Special cases

1. When $\delta = 0, 2\pi, 4\pi, \cdots$, i.e., $\delta = 2m\pi$, $m = 0, 1, 2\cdots$, we have $\sin\delta = 0$ and $\cos\delta = 1$. $\therefore$ From (15.14.3), we obtain

$$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0 \Rightarrow \frac{x}{a} - \frac{y}{b} = 0$$

$$\therefore \quad \boxed{y = \frac{b}{a}x} \qquad (15.14.4)$$

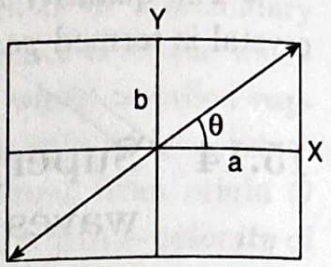

Fig. 15.24
Linearly polarised light with a positive slope

This is an equation of *a straight line through the origin with a positive slope* b/a. So, the resultant emergent light will be ***plane polarised*** *with vibrations in the same plane as the original incident light* (Fig. 15.24).

2. When $\delta = \pi, 3\pi, 5\pi, \cdots$, i.e., $\delta = (2m+1)\pi$, $m = 0, 1, 2, \cdots$, we have $\sin\delta = 0$, $\cos\delta = 1$. From (15.14.3), therefore,

$$\left(\frac{x}{a} + \frac{y}{b}\right)^2 = 0 \Rightarrow \frac{x}{a} + \frac{y}{b} = 0$$

$$\therefore \quad \boxed{y = -\frac{b}{a}x} \qquad (15.14.5)$$

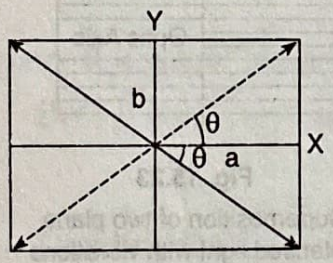

Fig. 15.25
Linearly polarised light with a negative slope

This again is the equation of a ***straight line*** *through the origin with a negative slope,* $-b/a$. So, the resultant emergent light is ***plane polarised***, the direction of vibration making an angle $-\theta$ with x-axis (Fig. 15.25).

3. When $\delta = \pi/2, 3\pi/2, 5\pi/2, \cdots$ i.e., $\delta = (2m+1)\frac{\pi}{2}$, $m = 0, 1, 2, \cdots$, $\sin\delta = 1$ and $\cos\delta = 0$. So, from (15.14.3)

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1} \qquad (15.14.6)$$

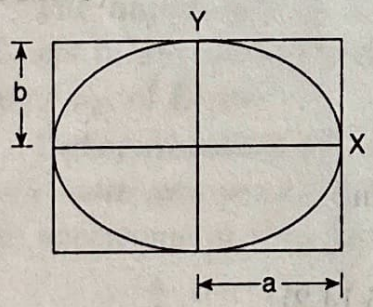

Fig. 15.26
Symmetrical elliptically polarised light

This is the equation of a ***symmetrical ellipse*** *with major and minor axes coinciding with the coordinate axes.* The emergent light will thus be ***elliptically polarised*** (Fig. 15.26), the axes of the ellipse being along and perpendicular to the optic axis.

4. When $\delta = \pi/2$ or $3\pi/2$ *and* $\theta = 45°$ i.e., the incident light making an angle $45°$ with the optic axis, then we have

$$a = A\cos 45°; \quad b = A\sin 45° \Rightarrow a = b.$$

So from (15.14.6),

$$\boxed{x^2 + y^2 = a^2} \qquad (15.14.7)$$

This is an equation of ***a circle*** and the emergent light will be ***circularly polarised.*** This is the basis on which a *quarter wave plate* works.

Summarising : In polarised light, *if the light vector*

(a) *vibrates along a straight line, it is a* ***plane or linearly polarised light,***

(b) *rotates along a circle, i.e., does not change its magnitude while in rotation, it will be* ***circularly polarised,***

(c) *rotates along an ellipse, i.e., changes its magnitude during rotation, it will be* ***elliptically polarised.***

• **An important note**—The elliptically polarised light and the circularly polarised light may again be both ***left-handed*** or *anticlockwise* and ***right-handed*** or *clockwise, according as the rotating light vector rotates anticlockwisely* or *clockwisely.*

For instance, when $\delta = \pi/2, 5\pi/2, 9\pi/2, \cdots$, the equation of motion of E- and O-waves become

$$\left.\begin{aligned} x &= a\cos\omega t \\ y &= b\sin\omega t \end{aligned}\right\} \qquad (15.14.8)$$

using the equations (15.14.1) and (15.14.2).

And the resulting elliptic vibration ($z = 0$ is the plane of observation) becomes

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

But the equations (15.14.8) show that this ellipse is traced by the tip of the light vector *rotating anticlockwise* (Fig. 15.27a), relative to an observer towards whom the light travels and the light is said to be ***left-handed elliptically polarised*** (*LEP*). Note that $\vec{r} = \hat{i}x + \hat{j}y$ rotates *counterclockwise* with increasing time.

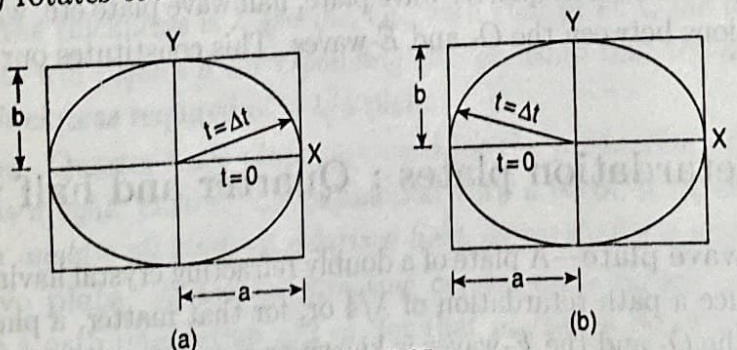

Fig. 15.27

Elliptically polarised light : (a) left-handed, (b) right-handed

If however $\delta = 3\pi/2, 7\pi/2, 11\pi/2, \cdots$ the equations for E- and O-waves are obtained from equations (15.14.1) and (15.14.2) as

$$\left.\begin{aligned} x &= -a\cos\omega t \\ y &= b\sin\omega t \end{aligned}\right\} \qquad (15.14.9)$$

and lead again to an elliptically polarised light, since here also

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

But the ellipse is *described clockwise* (Fig. 15.27b) relative to an observer towards whom light travels and the light is ***right-handed elliptically polarised*** (*REP*). Here $\vec{r} = \hat{i}x + \hat{j}y$ rotates *clockwise* with increasing time.

Note that half of the allowed δ-values for elliptic polarisation, as given in **special case 3**, correspond to *LEP* and the other half to *REP*.

In a similar fashion, there arises the *left-handed* and the *right-handed* ***circularly polarised light.*** For instance, over and above the two conditions just cited (i.e., $\delta = \pi/2$ and $\delta = 3\pi/2$), if $\theta = 45°$, we shall get in the first case : $a = b$ and the equations for E- and O-waves are

$$x = a\cos\omega t, \quad y = a\sin\omega t \tag{15.14.10}$$

and the resultant vibration becomes : $x^2 + y^2 = a^2$, representing a ***circularly polarised*** wave. But the *circle is described anticlockwise* relative to an observer to whom the light travels. So, the light is ***left-handed circularly polarised*** (*LCP*).

In the second case, $a = b$ and the equations for E- and O-waves are

$$x = -a\cos\omega t; \quad y = a\sin\omega t \tag{15.14.11}$$

which again leads to a *circle* : $x^2 + y^2 = a^2$, but the *circle is* now *described clockwise* relative to an observer to whom the light travels and the light is ***right-handed circularly polarised*** (*RCP*).

• We shall later see *how to convert a left handed elliptically* or *circularly polarised light into a right-handed one and conversely.*

In this context, we shall have to use a class of optical devices, called ***retarders*** or ***retardation plates*** such as quarter wave plate, half wave plate etc. which will introduce path retardations between the O- and E-waves. This constitutes our subject matter in what follows.

15.15 Retardation plates : Quarter and half wave plates

A. Quarter wave plate—A plate of a doubly refracting crystal having just a thickness so as to produce a path retardation of $\lambda/4$ or, for that matter, a phase retardation of $\pi/2$ between the O- and the E-waves is known as a ***quarter wave plate*** or ***$\lambda/4$-plate.***

Computation of thickness—Consider a plane parallel plate obtained by cutting a doubly refracting crystal with faces parallel to the optic axis. Let a beam of light of wavelength λ be incident normally on it (Fig. 15.28). The beam breaks up into O- and E-waves, both travelling along the same path perpendicular to the faces, but with different velocities.

For ***negative*** crystals (e.g., calcite), E-waves move faster than O-waves so that $n_0 > n_e$, where n_0 is the principal index of the crystal for O-wave, n_e that of the

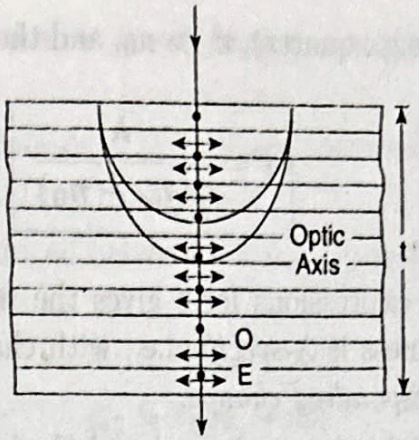

Fig. 15.28
Quarter wave plate : Thickness computation

E-waves. If t be the thickness of the plate, the path difference between the two waves, on emergence, is

$$\Delta = (n_0 - n_e)t$$

For the plate to be a quarter wave plate the path difference $\Delta = \lambda/4$,

$$\therefore \quad (n_0 - n_e)t = \lambda/4$$

$$\Rightarrow \quad \boxed{t = \frac{\lambda}{4(n_0 - n_e)}} \qquad (15.15.1)$$

For a ***positive*** crystal (e.g., quartz), $n_e > n_0$, so that

$$\boxed{t = \frac{\lambda}{4(n_e - n_0)}} \qquad (15.15.2)$$

So, a plate of thickness given by either (15.15.1) or (15.15.2) will serve as a *quarter-wave plate* for a given wavelength λ.

• Obviously, the thicknesss is λ-*specific*, i.e., with *change in* λ, *the thickness* of the quarter wave plate will *require a corresponding change*. Note that the above t-value is the ***minimum thickness*** required of a $\lambda/4$ plate.

Applications—Quarter wave plates are used for the *production of circularly and elliptically polarised light*. Further, in conjunction with a Nicol, it becomes a versatile analyser and can *analyse all kinds of polarised light*, as we shall see later.

B. Half-wave plate—A doubly refracting crystal plate having just a thickness so as to produce a path retardation $\lambda/2$ or, for that matter, a phase retardation of π between the *O*- and *E*-waves is called a ***half-wave plate*** or ***$\lambda/2$-plate.***

Computation of thickness—Let t be the thickness of a half-wave plate. Then by definition, we have for a ***negative*** *crystal* (i.e., calcite), $n_0 > n_e$, so that

$$(n_0 - n_e)t = \lambda/2$$

$$\Rightarrow \quad \boxed{t = \frac{\lambda}{2(n_0 - n_e)}} \qquad (15.15.3)$$

15.17 Production of circularly and elliptically polarised light

The methods of the prodution of both circularly and elliptically polarised lights are described hereunder.

A. Circularly polarised light : A *circularly polarised* light can be produced by superposing two coherent linear vibrations, of equal amplitudes, at right angles to each other with a phase difference of $\pi/2$ or a path difference of $\lambda/4$. This is done with the help of a *quarter wave plate.* For instance,

$$\left.\begin{aligned} x &= a\cos(\omega t + \pi/2) \\ \text{and} \quad y &= a\cos\omega t \end{aligned}\right\}$$

give, on their superposition, a *circular* vibration : $x^2 + y^2 = a^2$.

The ***experimental set up*** is shown in Fig. 15.30. A parallel beam of monochromatic light is allowed to fall on Nicol N_1 so that the emergent light is plane polarised. N_2 is a second Nicol *at cross* with N_1 so that *no light is transmitted* by N_2.

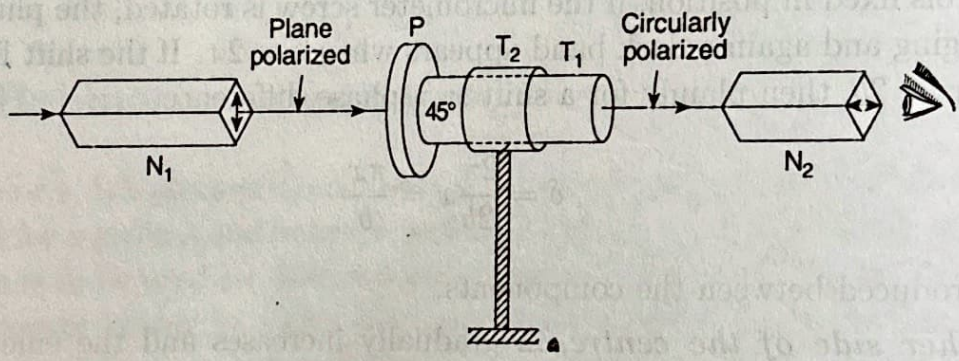

Fig. 15.30
Experimental set up for circularly and elliptically polarised light

Now, a $\lambda/4$-plate, mounted on a tube T_1, the circumference of which is graduated in degrees, is inserted between N_1, N_2 and held *normal* to the incident beam. Tube T_1 can rotate in an outer fixed tube T_2 mounted on a stand so that the $\lambda/4$ plate can rotate about a horizontal axis through any desired angle. With $\lambda/4$-plate inserted, some light may appear in the field of view of N_2. The $\lambda/4$-plate is next rotated till the field of view of N_2 is again *dark.* This ensures that *vibrations of light incident on $\lambda/4$-plate are parallel to its optic axis* and both E- and O-waves travel in the same direction but with different velocities to be refused transmission through N_2 *at cross* with N_1. The $\lambda/4$-plate is now rotated through 45° with its optic axis so that the amplitudes of E- and O-waves are equal and also a *phase difference* of $\pi/2$ is introduced between the two waves. The resulting beam emerging out of the $\lambda/4$-plate will therefore be ***circularly polarised*** (CP).

B. Elliptically polarised light—An elliptically polarised light can be produced by superposing two coherent linear vibrations of *unequal amplitudes,* at right angles to each other, with a phase difference of $\pi/2$. The help of a $\lambda/4$-plate is utilized here also.

For instance, the linear vibrations

$$\left.\begin{aligned} x &= a\cos(\omega t + \pi/2) \\ \text{and}\quad y &= b\cos\omega t \end{aligned}\right\}$$

on their superposition, yield an elliptic vibration :

$$x^2/a^2 + y^2/b^2 = 1.$$

The ***experimental set-up*** (Fig. 15.30) is the same as in the production of circularly polarised light. A parallel monochromatic light beam is allowed to fall on Nicol N_1. Next, the emerging plane polarised light falls on a second Nicol N_2, *at cross* with N_1. So the field of view of N_2 is *dark* . The $\lambda/4$ plate P is now inserted between N_1, N_2 and held normal to the incident beam. The field of view of N_2 may now show some light. The $\lambda/4$ plate is now rotated till the field view of N_2 is dark again. In this position, the vibrations of incident light on $\lambda/4$-plate are $\parallel$ optic axis of the plate and so perpendicular to N_2.

The quarter wave-plate is again rotated so that the vibrations of the incident light on it make any angle ***other than*** $45°$. *This would ensure unequal amplitudes of the extra-ordinary and the ordinary rays and the light from the* $\lambda/4$*-plate would be* ***elliptically polarised.***

15.18 Detection of circularly and elliptically polarised light

The methods of detection of circularly and elliptically polarised lights are rather simple and are described in what follows.

Circularly polarised light—The circularly polarised light, when seen through a rotating Nicol, shows no variation in intensity and thus resembles unpolarised light. So, to detect if the given light is circularly polarised, it is first passed through a $\lambda/4$-plate with its *optic axis in any position.*

The light that will emerge out of the $\lambda/4$-plate will become *plane polarised.* This plane polarised light is next examined with a Nicol prism. If the Nicol, on rotation, shows a variation in intensity with *zero minimum*, then the original light under test is ***circularly polarised.***

Elliptically polarised light—To detect if the light emerging out of $\lambda/4$ plate is elliptically polarised or not, it is first examined with a rotating Nicol which is adjusted to the position of *maximum intensity.*

Next, another $\lambda/4$-plate is interposed between the first $\lambda/4$-plate and the analysing Nicol such that *its optic axis is parallel to the principal section* of the Nicol, adjusted for *maximum intensity.* If the light is *elliptically polarised*, it will now become *plane polarised* and on rotating a Nicol, there will be a variation in intensity between the *maximum* and *zero minimum.*

15.19 Conversion of one type of polarised light into another

We shall now discuss below the various ways by which one type of polarised light may be converted into another type.

1. Elliptically polarised light → plane polarised light → circularly polarised light—Insert a $\lambda/4$-plate with the optic axis parallel to the principal section of an analysing Nicol adjusted for maximum intensity. Then the optic axis of the $\lambda/4$ plate is $\parallel$ to the major axis of the elliptic vibration. In this situation, the $\lambda/4$-plate introduces a further phase difference between the two vibrations at right angles to each other. The light emerging from $\lambda/4$-plate becomes ***plane polarised.***

Now, ***to convert the plane polarised light into circularly polarised light***, allow it to fall normally on another $\lambda/4$-plate such that the direction of vibration of plane polarised light makes 45° with the direction of the optic axis of $\lambda/4$-plate. The $\lambda/4$-plate breaks the incident wave into E- and O-waves of equal amplitude and introduces a phase difference of $\pi/2$ between them. On emergence from the plate, the O- and E-waves combine to give ***circularly polarised light.***

2. A left-handed elliptically polarised light to a right-handed one and conversely—For this, a phase difference of π must be introduced. So it is allowed to be incident normally on a half-wave plate such that its vibrations make an angle θ, ***excluding*** **0°, 45°** and **90°**, with the direction of the optic axis. The emergent light will become ***right-handed elliptically polarised***, as the phase difference between the component waves (E and O) is $\delta = \pi/2 + \pi = 3\pi/2$.

3. A left-handed circularly polarised light into a right-handed one and conversely—A phase difference of π must be introduced. So it is allowed to pass normally through a half-wave plate such that its vibrations make an angle of 45° with the optic axis. The emergent light from the $\lambda/2$-plate will be ***right-handed circularly polarised light***, as the phase difference between the component waves is $\delta = \pi/2 + \pi = 3\pi/2$.

4. A circularly polarised light into a plane polarised light—If a circularly polarised light is passed through a $\lambda/4$-plate such that its vibrations make an angle of 45° with its optic axis, the light emerging from the plate will be ***plane polarised.*** This is because the phase difference between the components (E and O) becomes $\delta = \pi/2 + \pi/2 = \pi$.

15.20 Applications of Babinet's compensator

Of the different applications of Babinet compensator, we shall discuss only two of them.

1. Production of elliptic and circularly polarised light—The prism B is shifted by $x = b/2$ from its position of coincidence with prism A; then the phase difference introduced is $\pi/2$ (a shift $2b$ produces a phase difference of 2π). If the direction of vibration of the incident plane polarised light (obtained from a polariser)

makes an angle α ($\neq 45°$) with the optic axis of A, then the emergent light will consist of *two rectangular* vibrations of *unequal amplitude* and *differing in phase* by $\pi/2$. The result is ***elliptically polarised*** light.

If however $\alpha = 45°$, the two rectangular vibrations will have *equal amplitude* and the emergent light will be ***circularly polarised.***

2. Analysis of elliptically polarised light—This entails (a) *determination of the phase difference between the two vibrations* giving rise to the elliptic light and (b) the *determination of the positions and ratio of the axes* of the elliptic light.

(a) First, the compensator is inserted between two *crossed* Nicols. Using white light and rotating the micrometer screw, the central dark band is made to coincide with the cross-wire of the eyepiece. Polariser is then removed and the compensator is allowed to receive the elliptic light under test. The central dark band gets shifted and is brought back to its original position by shifting the screw through x.

$$\therefore \quad \text{Required phase difference, } \phi = (\pi/b)x,$$

$2b$ being the shift corresponding to a phase difference by 2π.

(b) First, the compensator is inserted between two *crossed* Nicols. Using white light and rotating the micrometer screw, the central dark band is made to coincide with the cross-wire of the observing eyepiece. The micrometer screw is then turned through $b/2$ to introduce a phase difference of $\pi/2$ at the centre.

The central dark band is no more on the cross-wire. Polariser is now removed and compensator illuminated by elliptically polarised light under test. The compensator is *rotated in its own plane* till the central dark band is in the cross-wire. Analyser is also rotated to make the bands distinct. *The axes of the elliptically polarised light are now parallel to the optic axis of the wedges of the compensator* (Fig. 15.31).

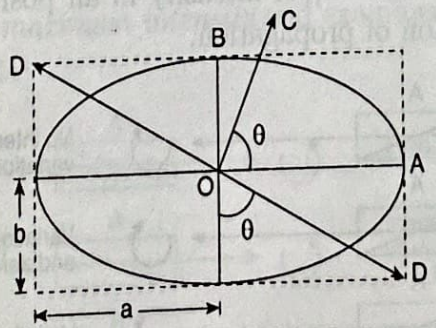

Fig. 15.31
Analysis of elliptically polarised light

The situation, as shown in Fig. 15.31, indicates that OA and OB are the optic axes of the wedges, OC the direction of principal section of analyser and DD' the direction of light emerging from the compensator at the cross-wire. Plainly, the *tangent of the angle θ made by the principal section of analyser with the optic axis of the wedge gives the ratio of the axes* :

$$\tan\theta = (b/a)$$

To determine θ, the analyser is rotated until the bands disappear to give uniform illumination. The angle of rotation is θ.

15.21 Scheme for a complete analysis of polarised light

According to a theorem due to Stokes, only *seven* possible states of polarisation of a beam of light can occur. These are as under :

1. ***unpolarised or natural light,***
2. ***plane or linearly polarised light,***
3. ***elliptically polarised light,***
4. ***circularly polarised light,***

and the rest *three* are ***mixtures*** each of unpolarised and any of the other three.

On passing the light through an analyser, e.g., a Nicol, that can rotate about the direction of light as axis, one can detect *elliptically* or *plane polarised* light since the intensity of the transmitted light varies with the orientation of the analyser. But if there is *no variation with orientation* then the original light may be *either unpolarised* or *circularly polarised* or *a mixture of the two.*

If however the Nicol is used *in conjunction with a quarter-wave plate*, one can make ***a complete, systematic*** *qualitative analysis of light.* The procedure is as follows.

Test I. Using a rotating Nicol – Insert the Nicol and rotate it. There are *three* possibilities.

(a) The *intensity* of the transmitted light is *independent* of the *orientation* of the analyser. This indicates that the light is either *unpolarised* or *circularly polarised* or *their mixture.* For, unpolarised light has vibrations of equal amplitudes in all directions and the rotating Nicol cannot show any variation of intensity. And circularly polarised light is the resultant of two mutually perpendicular vibrations of equal amplitudes and the rotating Nicol transmits equal intensity in all positions due to symmetry of vibrations about the direction of propagation.

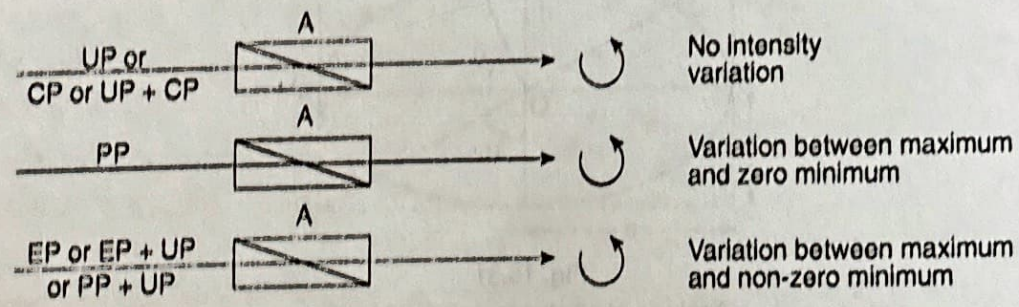

Fig. 15.32
Test I : Pictorial representation

(b) The intensity of transmitted light varies and *no light* is transmitted *for one* particular *orientation* of the analyser. This indicates that the light is *completely plane polarised.* For, plane polarised light has vibrations only in one direction and when the direction is perpendicular to the principal section of Nicol, Nicol stops transmission.

(c) The *intensity* of transmitted light *varies* with the orientation of the analyser but *never falls to zero.* This indicates that the light is either *elliptically polarised* or

a mixture of elliptically polarised and unpolarised light or *a mixture of plane polarised and unpolarised light.*

Fig. 15.32 represents the situations pictorially.

Test II. To investigate I(b), the light is first passed normally through a $\lambda/4$-plate followed by an analyser which is rotated. Again, there are the *same three possibilities.*

(a) If the *intensity* is *independent of orientation* of the analyser, it indicates that the given light is *unpolarised.*

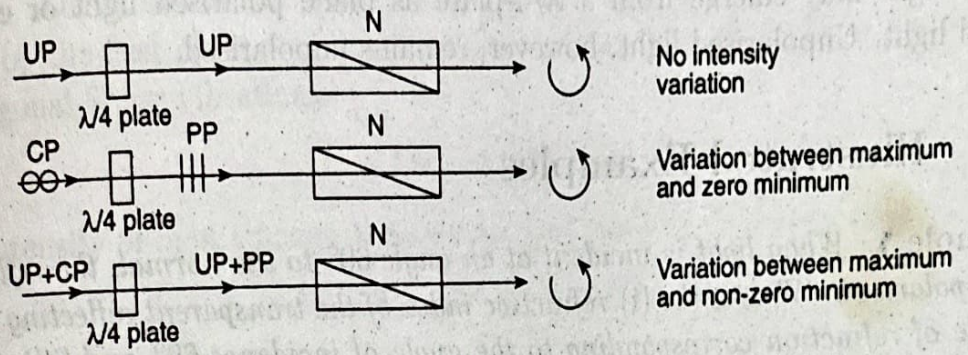

Fig. 15.33
Test II : Pictorial representation

(b) If there is *no transmission* of light for *one orientation of analyser*, then the original beam is *circularly polarised.*

(c) If the *intensity varies*, but *never falls to zero*, then it is a *mixture of circularly polarised* and *unpolarised light.*

Fig. 15.33 pictorially represents the situations.

Test III. To investigate I(c) further, again a $\lambda/4$-plate and an analyser are used, but now the $\lambda/4$-*plate is placed with is optic axis parallel to the principal section of the analyser adjusted for maximum intensity* and examined by rotating the Nicol. The *possibilities* are (Fig. 15.34) :

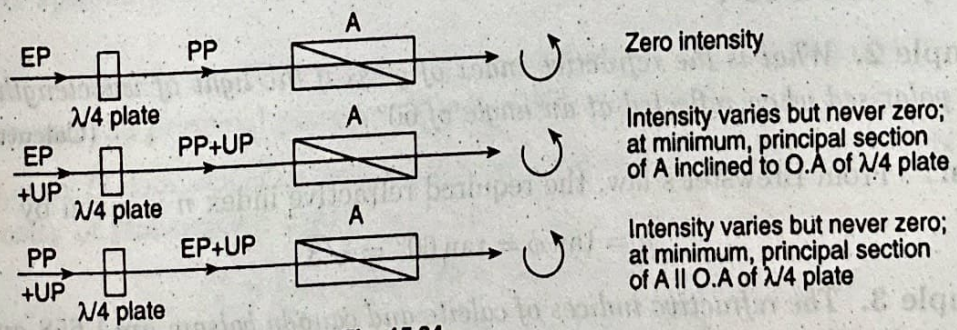

Fig. 15.34
Test III : Pictorial representation

(a) If the *light is extinguished for one setting of the analyser*, it indicates that the given light is *elliptically polarised.*

(b) If the *light-intensity varies but never falls to zero* and the position of *maximum* intensity *occurs at the same setting* of Nicol, it indicates that the light is a *mixture of plane polarised and unpolarised light.*

Solution : The incident light moves in denser medium. If n be the refractive index of the rarer (second) medium and c the critical angle, then

$$n = \sin c = \sin 45° = 1/\sqrt{2} = 0.707$$

If now ϕ be the polarising angle, then from Brewster's law, we obtain

$$\tan \phi = n \;\Rightarrow\; \phi = \tan^{-1} n = \tan^{-1}(0.707) = \mathbf{35.26°}$$

► **Example 8.** *At what angle β above the horizon is the sun when a person observing its rays reflected in water ($n = 1.33$) finds them linearly polarised along the horizontal?* **(North Bengal Hons.)**

Solution : Polarisation by reflection is dictated by Brewster's law : $\tan \phi = n$.

$\therefore\ \phi = \tan^{-1} n = \tan^{-1} 1.33 \simeq 57°$. Hence $\beta = 90° - 57° = \mathbf{33°}$.

► **Example 9.** *A thin polaroid is interposed between a pair of two crossed polaroids and made to rotate at a rate ω about their common axis. If an unpolarised light of intensity I_0 falls on the first polaroid, then show that the transmitted intensity is*

$$I = \frac{I_0}{16}(1 - \cos 4\omega t)$$

Solution : On transmission through the first polaroid, the intensity is $\frac{1}{2} I_0$ where I_0 is the intensity of unpolarised light. At any instant t, the angle between the axes of the first and the middle polaroid is $\theta = \omega t$.

$\therefore$ By Malus law, the intensity on transmission through the middle polaroid is

$$I_1 = \frac{I_0}{2} \cos^2 \theta = \frac{I_0}{2} \cos^2 \omega t$$

The angle between the axes of the middle and the last polaroid $= 90° - \theta$. So, the intensity of the transmitted light is :

$$I = I_1 \cos^2(90° - \theta) = \frac{I_0}{2} \cos^2 \omega t \sin^2 \omega t$$

$$= \frac{I_0}{8} \sin^2 2\omega t = \frac{I_0}{16}(1 - \cos 4\omega t)$$

► **Example 10.** *Calculate the thickness of $\lambda/4$-plate for light of $\lambda = 5893$ A, given $n_0 = 1.544$ and $n_e = 1.553$.* **(Calcutta Hons; Himachal Univ.)**

Solution : Here $\lambda = 5893 \times 10^{-10}$ m, $n_0 = 1.544$ and $n_e = 1.553$. Since $n_e > n_0$, the $\lambda/4$-plate is of a *uniaxial positive crystal.*

$$\therefore \text{ Thickness, } t = \frac{\lambda}{4(n_e - n_0)} = \frac{5893 \times 10^{-10}}{4(1.553 - 1.544)} = \frac{5893 \times 10^{-10}}{4 \times 0.009}$$

$$= \mathbf{0.1637 \times 10^{-4}\ m}$$

▶ **Example 11.** *Calculate the thickness of a half-wave plate for sodium light, given* $n_0 = 1.54$ *and the ratio of the velocities of O-component and E-component is* 1.007. *Is the crystal positive or negative?* **(Poona Univ.)**

Solution: Here $\lambda = 5893 \times 10^{-10}$ m; $n_0 = 1.54$. Also, by the problem $v_0/v_e = 1.007 \Rightarrow n_e/n_0 = 1.007$ where v's are velocities.

$$\therefore \quad n_e = 1.007 n_0 = 1.007 \times 1.54 = 1.551$$

Since $n_e > n_0$, the crystal is ***positive***.

Thickness of $\lambda/2$-plate is given by

$$t = \frac{\lambda}{2(n_e - n_0)} = \frac{5893 \times 10^{-10}}{2(1.551 - 1.540)} = \mathbf{0.2679 \times 10^{-4}\ m}$$

▶ **Example 12.** *A beam of linearly polarised light is changed into a circularly polarised light by passing it through a slice of crystal* 0.003 cm *thick. Find the difference in the indices of the two types of rays in the crystal, assuming it to have the minimum thickness that will produce the effect* ($\lambda = 6 \times 10^{-7}$ m). **(Calcutta H.; Indian Adm. Service)**

Solution: Here $\lambda = 6 \times 10^{-7}$ m; $t = 0.003$ cm $= 3 \times 10^{-5}$ m. Now, since a linearly polarised light changes into a circularly polarised one, the crystal plate has a thickness equal to that of a quarter wave plate.

$$\therefore \quad t = \text{minimum plate thickness} = \lambda/4(n_e \sim n_0)$$

$$\therefore \quad n_e \sim n_0 = \frac{\lambda}{4t} = \frac{6 \times 10^{-7}}{4 \times 3 \times 10^{-5}} = \mathbf{0.005}$$

▶ **Example 13.** *A sheet of cellophane is a half-wave plate for light of* $\lambda = 4 \times 10^{-5}$ cm. *Assuming that there is negligible variation in indices of refraction with* λ, *how would the sheet behave with respect to light of wavelength* 8×10^{-5} cm?

Solution: If t be the thickness of the half-wave plate, we have

$$t = \frac{\lambda}{2(n_e \sim n_0)} = \frac{4 \times 10^{-5}}{2(n_e \sim n_0)} = \frac{8 \times 10^{-5}}{4(n_e \sim n_0)}$$

which corresponds to the thickness formula for a quarter wave plate of $\lambda = 8 \times 10^{-5}$ cm. So, for $\lambda = 8 \times 10^{-5}$ cm, the sheet would behave as a ***quarter-wave plate***.

▶ **Example 14.** *A beam of plane polarised light of* $\lambda = 6000$ A *is incident on a thin quartz-plate cut with faces parallel to the optic axis. Calculate the minimum thickness of the plate which introduces a phase difference of* 60° *between the O-rays and the E-rays. Given* $n_e = 1.553$ *and* $n_0 = 1.544$.

Review Questions

1. Distinguish between polarised and unpolarised light. Explain the terms : plane of polarisation and plane of vibration. **(Kurukshetra and Himachal Univ.)**

2. What is dichroism? How does it produce plane polarised light? Use a dichroic cyrstal to analyse plane polarised light. **(Punjab Univ.)**

3. Explain the method of producing plane polarised light by reflection. **(Benares and Guru Nanak Univ.)**

4. State and derive Brewster's law. What does the law become when the rays of light travel from a denser to a rarer medium? **(Himachal Univ.)**

5. Show, by Brewster's law, that light incident on a transparent substance at polarising angle gives reflected and refracted rays at right angles to each other. **(Calcutta and Burdwan Hons.)**

6. What is birefringence? How does it explain double refraction? Give some applications of birefringence. **(Guru Nanak and Punjab Univ.)**

7. What do you understand by 'double refraction'? What are ordinary and extraordinary rays? How will you show that these are plane polarised?

8. Define 'optic axis', and the 'principal section' of a crystal. What is an uniaxial crystal? What do you mean by 'positive' and 'negative' crystals'? Give one example for each. What is the difference between the two?

9. Describe the construction and action of a Nicol prism. Explain how a Nicol prism is used to produce and analyse plane polarised light. **(Calcutta H.; Cochin Univ.)**

10. State and explain Malus law. How will you prove the law experimentally? **(Guru Nanak Univ.)**

11. Give Huygens' theory of double refraction. Draw diagrams and discuss double refraction through uniaxial crystals due to plane wave when the optic axis is (a) inclined to the upper face but lying in the plane of incidence, (b) parallel to upper face but lying in the plane of incidence, (c) parallel to the upper face but perpendicular to the plane of incidence and (d) perpendicular to upper face. **(Poona and Guru Nanak Univ.)**

12. Explain the method of finding the indices of ordinary and extraordinary rays. **(Punjab Univ.)**

13. What are polaroids? What do you mean by H-polaroid and K-polaroid? Give some of their uses. **(Kurukshetra and Himachal Univ.)**

14. What is meant by an elliptically and a circularly polarised light? Explain them with necessary theory. **(Himachal and Punjab Univ.)**

4. State and prove Brewster's law. State and explain Malus law.
5. Explain the terms : Optic axis, principal plane, uniaxial crystal, positive and negative crystals.
6. What is birefringence? Explain double refraction and ordinary and extraordinary rays.
7. What is dichroism? What are polaroids? Give some of their applications. What is meant by H-polaroid?
8. For maximum polarisation of light by reflection from a plane dielectric, what is the angle between the reflected and the refracted rays? **(Burdwan Hons.)**
9. Write a short explanatory note on Wollaston prism. **(Punjab Univ.)**
10. What would you conclude about the nature of light from polarisation? **(Calcutta H. and Guru Nanak Univ.)**
11. What is the greatest importance of polarisation? **(Guru Nanak Univ.)**
12. Give the difference between positive and negative crystals? **(Kurukshetra Univ.)**
13. Explain what do you understand by the principal refractive index. **(Punjab Univ.)**
14. A black dot is marked on a white piece of paper. It is then viewed through a calcite crystal from the top :
    (a) How many images are expected to be seen and why?
    (b) Is it possible to see one image only? Explain your answer. **(Mysore Univ.)**
15. A Nicol prism cannot be used for highly convergent and highly divergent beam. Explain why.
16. What do you mean by elliptically and circularly polarised light?
17. Give the theory and construction of a quarter wave plate.
18. How can you convert plane polarised light into a circularly polarised light?
19. Explain with a neat diagram, the working of a Babinet's compensator.
20. What is the nature of the emergent light when circularly polarised light is passed through (a) a quarter-wave plate and (b) a half-wave plate. **(Benares Hindu Univ.)**
21. How will you detect plane polarised light from partially polarised light? **(Cochin Univ.)**
22. Indicate the state of the emerging light in the following three cases :
    (a) a beam of plane polarised light is passed through a quarter-wave plate.
    (b) a beam of elliptically polarised light is passed through a quarter-wave plate.
    (c) a beam of plane polarised light is passed through a $\lambda/4$-plate such that its vibrations make an angle of $45°$ with the optic axis of the plate on incidence.

16

Optical Activity and Polarimeters

16.1 Introduction

Some crystals possess the characteristic property that when plane or linearly polarised light passes through them along the optic axis, they rotate, as the beam propagates, the plane of polarisation through a certain angle about the direction of propagation. This phenomenon is termed *rotatory polarisation* or ***optical activity***. The angle of rotation depends on a number of factors : (i) the thickness of the crystal, (ii) its nature and (iii) the wavelength of the light used. The substances that rotate the plane of polarisation are called ***optically active***.

There are *two* types of optically active substances : *dextro-rotatory and leavo-rotatory*. Those which rotate the plane of polarisation *clockwise*, while looking *against* the direction of light, are called ***dextro-rotatory*** or ***right-handed*** while those that rotate the plane of polarisation *anticlockwise*, on looking *against* the light, are called ***laevo-rotatory*** or ***left-handed***.

The optical activity of a crystal is due to its structural dissymmetry. The atoms or the molecules consituting the optically active crystals are arranged on a spiral which may be left-handed or right-handed. Some of such crystals have both the dextro and leavo varieties, one being just the mirror image of the other. Example is afforded by quartz.

Some substances e.g., cane sugar, show in solution the optical activity. In such cases, the activity is associated with the structural asymmetry of the molecules themselves. There may be some preferential direction of rotation associated with a molecule. While the molecules in a solution are oriented at random, their rotations however do not cancel out since the sense of rotation is the same for molecules having the same type of dissymmetry. There are others where the optical activity is associated with both type of dissymmetry.

16.2 Experimental detection of optical activity

The experimental set-up for detection of optical activity is shown in Fig. 16.1. If a plane polarised light emerging from Nicol N_1 is examined through another Nicol N_2, *at cross* with N_1, the field of view is totally *dark*. Now a quartz plate C, cut with optic axis perpendicular to its face (so that there is no double refraction), be inserted between N_1 and N_2 such that light is incident normally on C, then some light passes

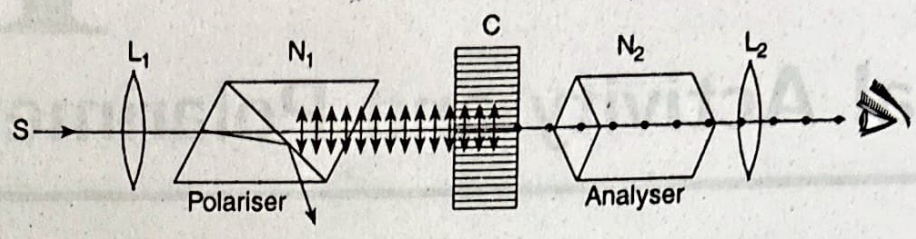

Fig. 16.1
Experimental set-up for detection of optical activity

through N_2. To cut off the light *completely* again, N_2 is to be rotated through a certain angle (whose value depends on the thickness of C). This proves that light emerging from quartz still remain plane polarised but its plane of polarisation has been rotated by the plate through a certain angle. Quartz is thus an *optically active* crystal.

16.3 Biot's studies : Specific rotation

Biot conducted systematically a series of experiments on optical rotation using different substances and came out with the following laws.

1. The *angle of rotation* θ of the plane of polarisation, for a given wavelength λ, is *directly proportional to the length* l of the active substance *traversed* by the polarised light and is not affected by turning the substance around. So the rotation will be annulled if the ray is reflected back through it.

2. For *solutions* and *vapours*, the *angle of rotation* θ for a given path length is *proportional to the concentration* of the solution and vapour.

3. The *combined rotation* produced by a *number of active substances* having different thicknesses is the *algebraic sum of the individual rotations*—the clockwise and the anticlockwise rotations are taken with opposite signs.

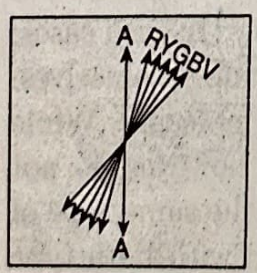

Fig. 16.2
Rotatory dispersion

4. The *angle of rotation* θ is (approximately) *proportional inversely* to the *square of the wavelength* λ. For quartz, we may write

$$\boxed{\theta = A + B/\lambda^2} \qquad (16.3.1)$$

where both A and B are constants.

This implies that if plane polarised white light with vibrations along AA (Fig. 16.2) be incident normally on a quartz plate, the

different colours will be rotated by different angles and the field of view will appear coloured. This phenomenon is called ***rotatory dispersion.***

5. *The angle of rotation also depends on the temperature* of the active substance.

In the light of the above characteristics of the phenomenon, we may now define a rather *important* quantity called the ***specific rotation*** or the ***rotatory power.***

Specific rotation—***The specific rotation S of an optically active substance at a given temperature T and for a given wavelength λ of light, is defined as the rotation (in degrees) produced by one decimeter[1] length of the substance in solution when its concentration is*** 1 g/c.c.

If θ be the rotation in degrees produced by l decimeter length of a solution and c be the concentration in g/c.c, then by the above definition,

$$[S]_\lambda^T = \frac{\theta}{l \times c} = \frac{\text{rotation in degrees}}{\text{length in decimetre} \times \text{concentration in g/c.c}}$$

$$\Rightarrow \boxed{\theta = S \times l \times c} \qquad (16.3.2)$$

- Note that c = simple density for pure liquids. If l be expressed in centimeter, then from the above equation (16.3.2), we have

$$\theta = S \times (l/10) \times c$$

- For optically active crystals, the *specific rotation is defined as the rotation produced by the crystal plate of thickness* 1 mm.

- The *product of the specific rotation and the molecular weight* of the active substance is known as ***molecular rotation.***

$\therefore$ ***Molecular rotation = specific rotation × molecular weight.***

16.3.1 An important theorem

Statement—***A linear vibration may be considered to be equivalent to two oppositely directed circular vibrations.***

Proof. Let $x = 0$ and $y = a\cos\omega t$ be a linear vibration along the y-axis in the xy-plane, a being the amplitude and $\omega/2\pi$ the frequency of vibration.

The above equations may be re-written as

$$x = \tfrac{1}{2}a\sin\omega t - \tfrac{1}{2}a\sin\omega t = x_1 + x_2$$

$$y = \tfrac{1}{2}a\cos\omega t + \tfrac{1}{2}a\cos\omega t = y_1 + y_2$$

and can be taken as the resultant of the following two vibrations (the principle of superposition) :

[1]The rotation due to a liquid is much less than that due to a crystal. So, the standard path length for a liquid is taken as 1 decimetre instead of 1 mm for crystals.

$$x_1 = \tfrac{1}{2}a\sin\omega t; \quad y_1 = \tfrac{1}{2}a\cos\omega t \quad \text{(16.3.3)}$$

$$x_2 = -\tfrac{1}{2}a\sin\omega t; \quad y_2 = \tfrac{1}{2}a\cos\omega t \quad \text{(16.3.4)}$$

Now, the two equations in (16.3.3) are the components of a *clockwise circular vibration* (since $x_1^2 + y_1^2 = (a/2)^2$) in two perpendicular directions. Equations in (16.3.4) similarly denote the components of *an anticlockwise circular vibration* along the same two directions. Since these are derived from a linear vibration, *the linear vibration is equivalent to two oppositely directed circular vibrations of the same frequency.*

16.4 Fresnel's theory of optical rotation

The Fresnel's theory of optical rotation is based on the principle (just proved) that a linear vibration is equivalent to the resultant of two oppositely directed (anticlockwise and clockwise) circular vibrations of the same frequency.

Assumptions—Fresnel made the following assumptions to develop his theory to explain the optical rotation.

1. The plane polarised light on entering a substance gets split into *two circularly polarised* waves—one clockwise and the other anticlockwise.

2. The above two waves move with the same speed in an optically non-active substance, while in active substances they travel with different speeds—the clockwise moves faster in the dextro-rotatory substance; in the laevo-rotatory substance, the anticlockwise one moves faster. This results in a *phase difference* being developed between them as they move in the substance.

3. On emergence, the *two circular components recombine* to produce the same *plane polarised* light with the difference that the plane of polarisation is rotated relative to the incident one by an angle depending on the phase difference.

Explanation—Let the plane polarised light fall normally on a quartz plate cut perpendicular to the optic axis, the first face being in xy-plane. Let the vibration be

$$y = a\cos\omega t$$

which, on just entering the crystal, splits into the following two equal and opposite circular motions (Fig. 16.3a) : OL is the circularly polarised vector rotating anti-clockwise (left-handed) and OR is the circularly polarised vector rotating clockwise (right-handed), OA being their resultant vector.

$$x_1 = \tfrac{1}{2}a\sin\omega t, \quad y_1 = \tfrac{1}{2}a\cos\omega t \quad \textit{(clockwise circular)}$$

$$x_2 = -\tfrac{1}{2}a\sin\omega t, \quad y_2 = \tfrac{1}{2}a\cos\omega t \quad \textit{(anticlockwise circular)}$$

These circular components move through the plate with *different* speeds so that, on emergence, a phase difference δ would be introduced between them. Assuming the clockwise component moves faster (Fig. 16.3b), the emergent circular components are :

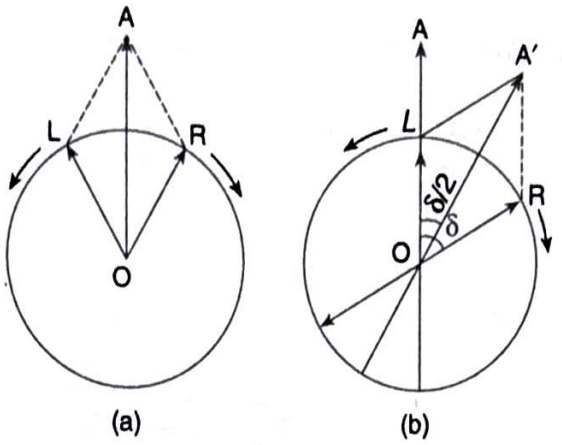

Fig. 16.3
Explanation of optical rotation : Fresnel's theory

$$\left.\begin{aligned} x_1 &= \tfrac{1}{2}a\sin(\omega t+\delta) \\ y_1 &= \tfrac{1}{2}a\cos(\omega t+\delta) \end{aligned}\right\} \qquad \left.\begin{aligned} x_2 &= -\tfrac{1}{2}a\sin\omega t \\ y_2 &= \tfrac{1}{2}a\cos\omega t \end{aligned}\right\}$$

On emergence, OR is the right-handed and OL the left-handed circularly polarised vectors with the resultant OA'.

$\therefore$ Resultant displacements along the axes are :

$$x = x_1 + x_2 = \frac{1}{2}a\,[\sin(\omega t+\delta) - \sin\omega t] = a\cos\left(\omega t + \frac{\delta}{2}\right)\sin\frac{\delta}{2} \qquad (16.4.1)$$

$$y = y_1 + y_2 = \frac{1}{2}a\,[\cos(\omega t+\delta) + \cos\omega t] = a\cos\left(\omega t + \frac{\delta}{2}\right)\cos\frac{\delta}{2} \qquad (16.4.2)$$

On dividing equation (16.4.1) by (16.4.2), we immediately obtain

$$\boxed{\frac{x}{y} = \tan\frac{\delta}{2}} \qquad (16.4.3)$$

which represents a *straight line inclined at an angle $\delta/2$ to the y-axis* (Fig. 16.3b). So, the emergent light from the plate is plane polarised with vibrations inclined at $\delta/2$ to the y-axis, i.e., relative to the incident vibrations.

Now, plainly, we have

$$\delta = \frac{2\pi}{\lambda} \times \text{path difference} = \frac{2\pi}{\lambda}(n_A - n_C)d \qquad (16.4.4)$$

where n_A and n_C are the refractive indices of quartz along the optic axis for the anticlockwise and the clockwise circular components respectively and d is the thickness of the crystal.

Rotation of the plane of polarisation is, therefore, given by

$$\boxed{\theta = \frac{1}{2}\delta = \frac{\pi}{\lambda}(n_A - n_C)d} \qquad (16.4.5)$$

16.6.1 Laurent half-shade polarimeter

The essential optical parts of a Laurent half-shade polarimeter are illustrated in Fig. 16.5a. Monochromatic light from a source (e.g., Na-vapour lamp), after passing through a narrow slit S, is rendered parallel by a converging lens L and allowed to

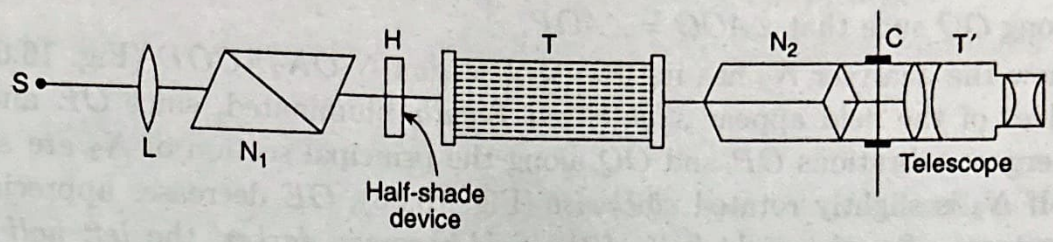

Fig. 16.5a
The Laurent half-shade polarimeter

fall on a polarising Nicol N_1. Light, emerging from N_1, is plane polarised and passes through the half-shade device H (called *Laurent's plate*) and thence through the glass tube T containing the active solution, onto the analysing Nicol N_2. The emerging light is viewed through the telescope T'. N_2 can be rotated about the light as axis (axis of tube) and its rotation in degrees measured by a vernier on the graduated circular scale C.

Working—Assuming H to be absent, the position of N_2 is adjusted till the field of view is totally dark, with tube T empty. In this position, N_2 is *at cross* with N_1. The position of N_2 is read. Next, T is filled with experimental solution and placed in position. This rotates the plane of polarisation of light from N_1 through some angle θ so that some light is now transmitted by N_2 and the field appears illuminated. N_2 is now rotated till the field is again dark and its reading noted. This happens when N_2 is rotated through θ in the direction of optical rotation. So, the difference of the two readings of N_2 gives the *rotation* of the plane of polarisation.

Action of half-shade—The difficulty in the above procedure is that the eye *cannot judge* the position of complete darkness accurately and the field appears dark rather over a wide range when the analyser N_2 is rotated.

To *remove this difficulty*, the half-shade device is inserted after the polarising Nicol N_1. It consists of two *semicricular plates* ACB and ADB (Fig. 16.5b) cemented along the dia AB. Plate ACB is of *quartz, cut parallel to optic axis*, while the plate ADB is of *glass*. The thickness of ACB is such that it introduces a phase difference of π between the O and E vibrations, i.e., it is a *half-wave plate*. The thickness of ADB is such that it absorbs the *same* amount of light as does the quartz plate.

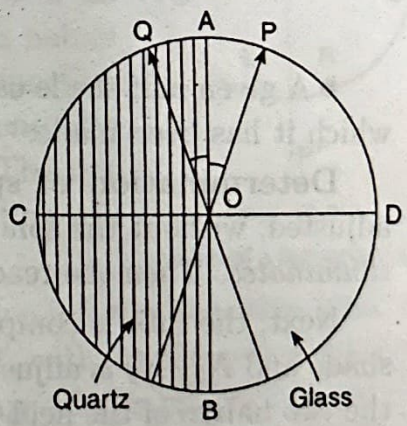

Fig. 16.5b
The half-shade

The plane polarised light from N_1 falls normally on half-shade device with vibrations along OP, say (Fig. 16.5b). Through the glass-half, it emerges as such with vibrations

still along OP. In the quartz-half, the light gets split into two—E-component parallel to the optic axis OA and the O-component perpendicular to the optic axis, i.e., along OD. In quartz, O-ray travels faster and gains, on emergence, a phase of π over E-ray. So, the O-component vibrates along OC (instead of OD) and E-component still having vibrations along OA. The resultant vibration of light emerging from quartz is thus along OQ such that $\angle AOQ = \angle AOP$.

If now the analyser N_2 has its *principal section* $N_2ON_2 \parallel COD$ (Fig. 16.6a), both the halves of the field appear *slightly* but *equally* illuminated, since OE and OF of the emergent vibrations OP and OQ along the principal section of N_2 are *small* but *equal*. If N_2 is slightly rotated *clockwise* (Fig. 16.6b), OE decreases appreciably and OF increases. So, the *right half of the field* becomes *darker*, the *left half brighter*. Converse is the case for slight *anticlockwise* rotation (Fig. 16.6c). The eye can judge more accurately the position of *equal illumination* of two halves than the position of complete darkness. So, the use of half-shade enables one to set N_2 more accurately.

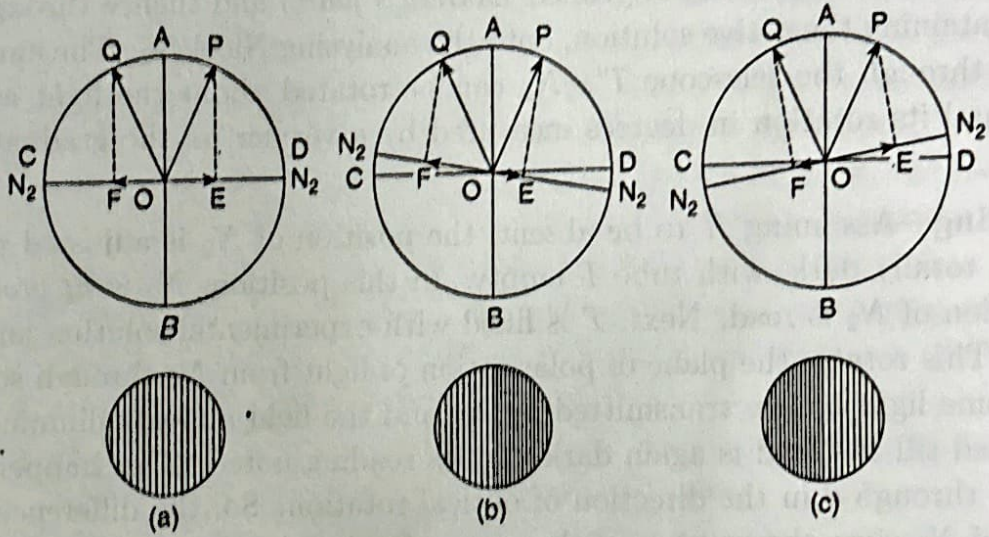

Fig. 16.6
Action of half-shade

• A given half-shade can be used for a particular wavelength only (λ-specificity) for which it has been made.

Determination of specific rotation of sugar solution—The analyser N_2 is adjusted, without the solution, for the two halves of the field to be slightly but *equally illuminated*. Then the reading on the circular scale is taken.

Next, the tube is completely filled with sugar solution and placed between the half-shade and N_2. N_2 is adjusted again and the reading for *small* but *equal illumination* of the two halves of the field is noted. The difference of the two readings gives the optical rotation θ caused by the given l decimeter long sugar solution. Then the specific rotation S is given by

$$(S)_T^{\lambda} = \frac{\theta}{l \times c}$$

where c is the concentration of the solution in g/c.c. and is assumed known.

Determination of strength of sugar solution—If m g of sugar is dissolved in V c.c. of an optically inactive solvent (e.g., water), then $c = m/V$. So, from the above equation,

$$S = \frac{\theta}{l \times (m/V)} \Rightarrow c = \frac{m}{V} = \frac{\theta}{S \times l}$$

where S = specific rotation for the same wavelength and temperature.

- The value of $(S)_T^{\lambda}$ may be obtained from the table of physical constants.

16.6.2 Bi-quartz polarimeter

It consists of a *narrow slit, a condensing lens, a polarising Nicol, bi-quartz plate, a tube* containing the optically active solution, an *analysing Nicol* and a *telescope* fitted with a *circular scale* graduated in degrees and a *vernier* just as in a half-shade polarimeter. The only difference is that the half-shade device is replaced here by a *bi-quartz plate* and the Na-*light by white light.* The basic polarimeter is thus identical with that of the half-shade. The arrangement is also the same. We shall therefore not describe the polarimeter but only explain the action of the bi-quartz plate.

Action of bi-quartz plate—It consists of *two similar semi-circular plates of quartz*—one *right-handed* (R) and the other *left-handed* (L), both *cut perpendicular to the optic axis* and cemented together along the diameter PP' and hence the name. The thickness of each plate is about 3.75 mm so that each rotates the plane of polarisation of yellow light through 90°, one clockwisely and the other anticlockwisely. When white light, made linearly polarised by the polarising Nicol, passes through the bi-quartz normally, it propagates along the *optic axis* and rotatory dispersion occurs in each plate. If the vibrations of incident light be along PP', then after the passage through the bi-quartz, the vibrations of yellow light (Y) are along a direction nromal to PP' and those of red (R) and blue light (B) are along different directions as shown in Fig. 16.7. If the principal section of the analysing Nicol be parallel to PP', the yellow light will get quenched in both halves of the field, but the red and blue will be present in equal proportion in each half. The two halves will then appear *equally illuminated* with a reddish-violet tint, called the *tint of the passage.* The tint is so sensitive that a slight rotation of Nicol from this position will make one half blue and the other half red.

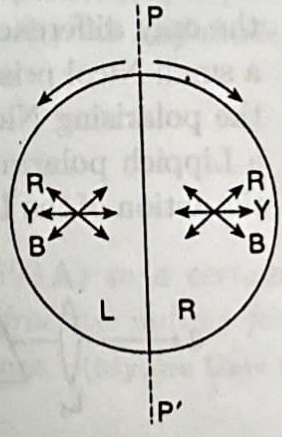

Fig. 16.7
Action of a biquartz plate

Determination of specific rotation—The position of the sensitive 'tint of passage' is obtained twice by rotating the analyser : First, without the active solution and secondly, with it. The difference of the two readings of the analyser gives the optical rotation θ caused by the active solution. Once θ is known, the specific rotation S can be obtained from the usual formula

$$S = \frac{\theta}{l \times c}$$

where l is the length of the solution in *decimeter* and c the concetration in g/c.c.

9. What do you mean by optical activity? Discuss the Fresnel's explanation of optical activity. **(Burdwan and Calcutta Hons.)**

10. Derive an expression for the angle by which the plane of polarisation of light of frequency ω is rotated in passing through a thickness d of an optically active substance. **(Calcutta Hons.)**

11. Calcite crystal is

(a) dextro-rotatory, (b) laevo-rotatory,
(c) both of these, (d) none of these.

12. The rotation of the plane of polarisation does *not* depend on

(a) wavelength of light, (b) temperature,
(c) thickness of the material, (d) any of these.

13. Of the following statements, which one is *not* correct?

(a) A quartz crystal may be right-handed.
(b) A quartz crystal may be left-handed.
(c) A quartz crystal may be both right- and left-handed.
(d) None of the above is true.

Problems

1. A 22 cm long tube containing 88 c.c. of sugar solution produces an optical rotation of 9.9° when placed in a polarimeter. If the amount of sugar in the solution is 6 g, find the specific rotation. [66°]

2. The specific rotation of quartz for $\lambda = 508.6\,\text{nm}$ is 29.73 deg/mm. Find the difference between the indices for the left and the right circularly polarised lights for quartz. $[8.4 \times 10^{-5}]$

3. A 20 cm long tube containing sugar solution gives a rotation of 11° of the plane of vibration of a plane polarised light. Find the strength of the solution. Given : sp. rotation of sugar $= 66°\,\text{dm}^{-1}\text{g}^{-1}\text{cm}^{-3}$. [83%]

4. A 20 cm column of cane sugar solution of concentration 100 g/litre gives an optical rotation of 10.6°. Find the purity of cane sugar. Given : sp. rotation of sugar $= 66.5°\,\text{dm}^{-1}\text{g}^{-1}\text{cm}^{-3}$. **(Calcutta Hons.)** [$\simeq$ 80%]

[**Hints.** Rotation by pure sugar solution, $\theta = 66.5 \times \frac{100}{1000} \times \frac{20}{10} = 13.3°$. With the given solution, the rotation is less, 10.6°, due to small % of sugar in solution. Now, the concentration is proportional to rotation. So the %-age of pure sugar in the solution is $\frac{100 \times 10.6}{13.3°} \simeq 80\%$]

5. A quartz crystal cut perpendicular to the optic axis gives a rotation of 20°/mm in the plane of polarisation of a plane polarised light. Calculate the thickness of a quartz plate which, when interposed between two parallel Nicols, would transmit no light. What would be the corresponding rotation? [4.5 mm; 90°]

6. 20 g cane sugar is dissolved in water to make a 50 c.c. solution. A 20 cm length of this solution causes an optical rotation of +53°55′. Calculate the specific rotation of the solution. **(North Bengal Hons.)**

7. A 200 mm long tube contains sugar solution. When it is placed in a saccharimeter, it gives an optical rotation of 11°. Find the strength of the solution. Given, the specific rotation of cane sugar = 66°. **(Kamraj Univ.)** [0.833 g/c.c]

8. A tube 20 cm long, filled with a solution containing 15 g of cane sugar per 100 c.c of water, is placed in the path of plane polarized light. Find the angle of rotation of the plane of polarisation. Specific rotation of sugar is 66°. **(Garhwal Univ.)** [19°48′]

9. A length of 10 cm of a 5% solution causes an optical rotation of 20°. How much length of a 10% solution of the same substance will cause a rotation of 35°? **(Calcutta Hons.)** [8.75 cm]

10. The rotation produced by quartz for plane polarised light, $\lambda = 5.461 \times 10^{-5}$ cm, is 5.54° mm^{-1} and for $\lambda = 4.047 \times 10^{-5}$ cm is 48.96° mm^{-1}. If in the visible region of the spectrum, this rotation is given by

$$\phi = a + b/\lambda^2$$

find the constants a and b. [-3.00 deg mm^{-1}; 851×10^{-10} deg mm^{-1} cm^{-2}]

11. A quartz plate, cut with its surface perpendicular to the optic axis, is required to annual completely the rotation of plane of polarisation of red light ($\lambda = 7600$A) by a 26.7 cm length of lactose solution (sp. rotation = 52.5°) containing 100 g of active substance per litre of the solution. What should be the thickness of the plate. Given : For $\lambda = 7600$A, $n_L = 1.53920$, $n_R = 1.53914$ for quartz. [0.9865 mm]